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MODELING ACTIVE BEACON COLLISION AVOIDANCE SYSTEM (BCAS) MEASUREMENT ERRORS: AN EMPIRICAL APPROACH

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16. Abstract

The Active Beacon Collision Avoidance System (BCAS) algorithm evaluation required the development of accurate input error models. This document describes a methodology used to model Active BCAS errors associated with the measurements of altitude and range. The models were required for use in the analysis of Active BCAS resolution performance in an error-degraded environment. A time series analysis of two independent sets of data from Active BCAS surveillance tracker test flights showed that Active BCAS measurement errors are highly sequentially correlated (autocorrelated) and time dependent. This indicated that dynamic (time dependent) models are more appropriate than the previous static (time independent) models. The analysis showed that range and altitude measurement errors could be characterized using first and second order autoregressive processes. The improvement in the characterization of the errors was achieved without increasing the model complexity. The number of parameters was increased by one compared to the number of parameters of the previous static models. With availability of more sets of data, not necessarily larger sets, the methodology could be applied to increase the accuracy of the parameter estimates. The methodology, described herein, could be applied to develop altitude and range measurement error models for other collision avoidance systems in which similar tracking procedures were used.

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METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

*1 in ≡ 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price $2.25, SD Catalog No. C13.10:286.

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F

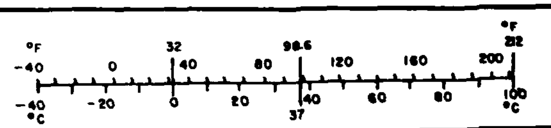

TABLE OF CONTENTS

Accession For
NTIS GRA&I ☑
DDC TAB ☐
Unannounced ☐
Justification
By
Distribution/
Availability Codes
Dist | Avail and/or special
A

LIST OF ILLUSTRATIONS

LIST OF TABLES

INTRODUCTION

PURPOSE.

This report presents an empirical approach to modeling Active Beacon Collision Avoidance System (BCAS) measurement errors, specifically, altitude measurement errors and range measurement errors. The analysis was conducted to study the characteristics of the BCAS aircraft (hereafter called own aircraft) altitude measurement error, intruder aircraft altitude measurement error and range measurement error, and to fit models to the available data. A secondary objective was to compare own and intruder altitude measurement errors.

SCOPE.

The fitted models and their parameter estimates described in this report are based on the analysis of two independent, but small, data bases. Therefore, the confidence regions for the parameter estimates are large. However, as will be shown later in this report, analysis of both sets of data resulted in highly consistent results.

More importantly, this report presents a methodology used to obtain the mathematical models of Active BCAS measurement errors. Once more data are available, the methodology could be used to increase the accuracy of the parameter estimates. The methodology could also be applied to develop altitude and range measurement error models of other collision avoidance systems in which similar tracking procedures are used.

BACKGROUND.

In previous efforts to identify the impact of measurement errors on Active BCAS performance (reference 1), Monte Carlo techniques were used to simulate measurement errors from static (time independent) models. The methodology presented in this report will permit the development of dynamic (time dependent) interactive error models, without increasing the model complexity. This approach provides a more direct means of evaluating the sequential impact of measurement errors on Active BCAS conflict resolution.

The two independent sets of data were obtained from Active BCAS surveillance test flights. The test flights were not designed to collect data to support error modeling. As a result, only a small part of the data included theodolite measurements. The theodolite measurements were required to accurately compute the errors.

Statistical tests indicated that the errors are time dependent and independent of their respective magnitudes of measurements. Thus, dynamic models are found to be more appropriate than previous static models. Throughout this report the terms "altitude error" and "range error" are used to mean "Active BCAS altitude measurement error" and "Active BCAS range measurement error," respectively. Likewise, the term "errors" is used to mean "measurement errors."

MODEL DEVELOPMENT

DATA BASES.

The analysis, described in this report, was limited by the amount of available data. The first data base resulted from Active BCAS flight tests conducted at the National Aviation Facilities Experimental Center (NAFEC), Atlantic City, New Jersey (reference 2). The set consisted of a continuous sequence of 63 discrete measurements, spaced at equal time intervals of 1 second, the update rate for Active BCAS. This set was collected when both own aircraft and intruder aircraft were in level flight. Throughout this report, this set of 63 data points is referred to as "level flight data." The experimental conditions under which the level flight data were collected are described in reference 2. The level flight data are included in appendix A.

The second data base consisted of 46 seconds of data collected at NAFEC and obtained from the MITRE Corporation (reference 3). These data permitted analysis of altitude and range error for a vertically maneuvering intruder. However, theodolite (true) position measurements were not available nor were own aircraft measurements. These were able to be estimated, however, since it was observed that constant vertical and range rates were maintained across the data collection period. This set of data is referred to as "climbing intruder data."

Previous studies have indicated that the transponder antenna structure may affect the performance of Active BCAS by reducing the data link reliability. The level flight data were obtained from flights in which both own and intruder aircraft had a top/bottom antenna structure. The antenna structure of the aircraft, which resulted in the climbing intruder data, was not reported.

For developing the error models, only the level-flight data were used. However, the climbing intruder data were also analyzed (appendix B) for comparison purposes. Analysis of the climbing intruder data resulted in findings consistent with the level-flight data analysis. The analysis of climbing intruder data was patterned after the level-flight data analysis and is included in appendix B.

"True" measurements (theodolite measurements) and BCAS surveillance measurements of own altitude, intruder altitude, and range were included in the level flight data (reference 2). The errors were computed by subtracting the BCAS surveillance measurements from the respective "true" measurements. Plots of altitude errors (own and intruder) are shown in figure 1, and a plot of range errors is shown in figure 2. The mean errors are also included in the figures.

The BCAS surveillance data analyzed represented data for established tracks. The Active mode BCAS surveillance tracker uses bracketing and altitude window search techniques to acquire tracks. Once the track altitude window is formed, replies falling inside the window are considered as an update to the altitude track. If no replies fall inside the window during the time interval, the altitude track is not updated for that time period. The BCAS surveillance tracking procedure is described in more detail in reference 2.

STATISTICAL ANALYSIS.

MEAN AND VARIANCE. The averages and variances of the level-flight data errors are presented in table 1. The own and intruder altitude information transmitted to

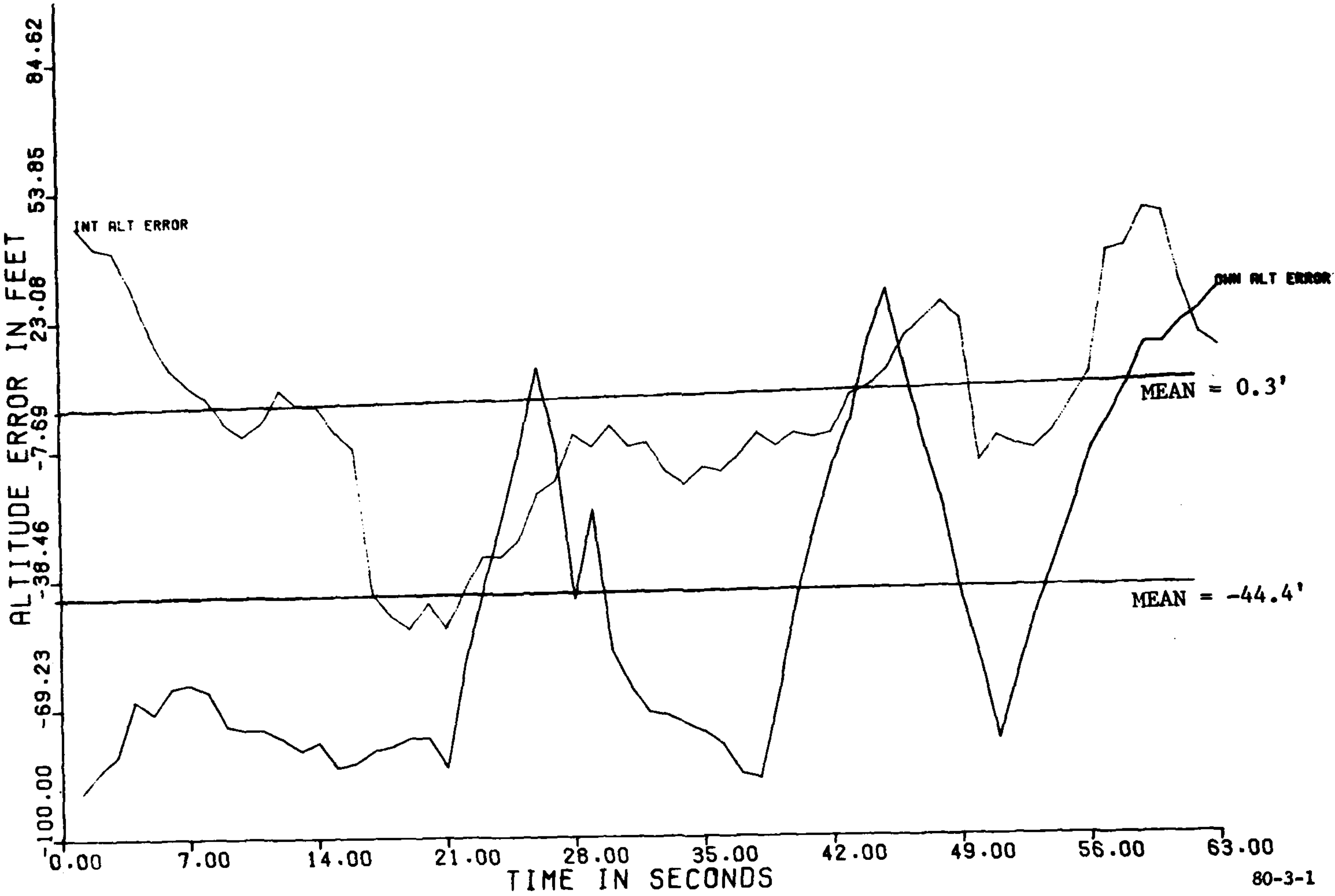

FIGURE 1. SEQUENTIAL OWN ALTITUDE AND INTRUDER ALTITUDE ERRORS—LEVEL FLIGHT

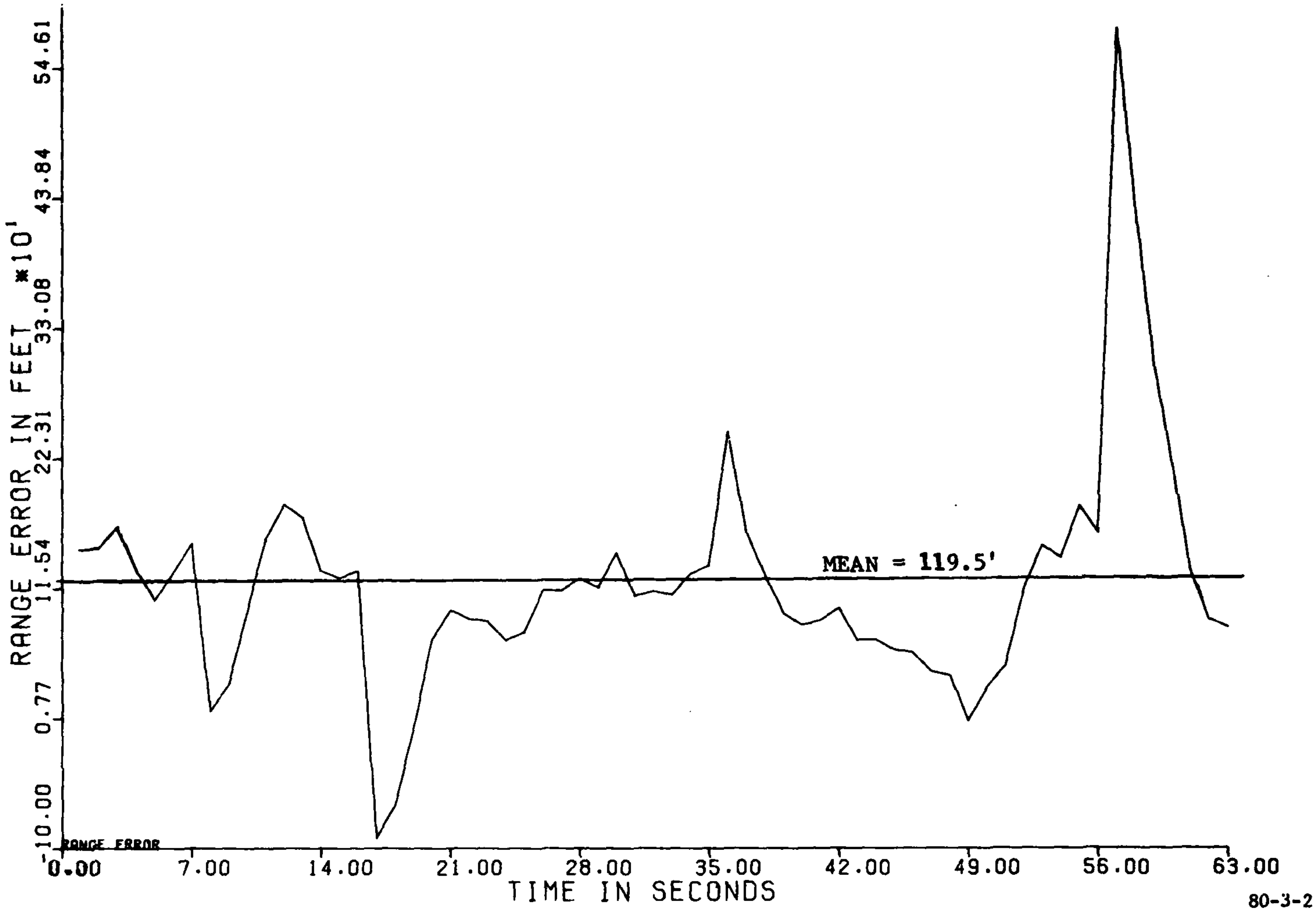

FIGURE 2. SEQUENTIAL RANGE ERROR—LEVEL FLIGHT

the BCAS are in the same form, encoded mode C data. Thus, one might expect the means and variances of their error sequences to be nearly equal. As can be seen in table 1, the own altitude error average (-44.4 feet (ft)) and variance (1,354.9 ft^2) are quite different from the intruder altitude error average (0.3 ft) and variance (579.6 ft^2).

TABLE 1. MEAN AND VARIANCE

Error	Average (ft)	Sample Variance (ft^2)	Standard Deviation (ft)
Own Altitude	-44.4	1,354.9	36.8
Intruder Altitude	0.3	579.6	24.1
Range	119.5	9,007.4	94.9

Investigation of the level-flight data (reference 2) identified a possible reason for the large bias in the own altitude error. Apparently, there was an interface problem between the own aircraft mode C encoder and the BCAS computer. The larger sample variance of own altitude error resulted because of the large peaked oscillations that occurred at t = 26, 36, 43, and 51 seconds. The peaks represent the quantization noise caused by the 100-foot granularity in mode C data. This quantization occurred when the own aircraft measured altitude deviated from the assigned altitude of 2,500 ft by more than 50 ft. Intruder altitude errors, however, as indicated by figure 1 and a review of raw data in reference 2, showed that the error pattern was not affected by mode C quantization noise. The maximum value of intruder altitude error (48 ft) occurred at t = 58 seconds. This indicated the intruder mode C reported altitude did not change during the data collection period.

The range measured was the slant range between aircraft. To offset the increase in signal turnaround time caused by the transponder reply delay, BCAS assumes an average transponder delay of 3 microseconds (μs). The average of the range errors (theodolite measurements minus BCAS measurements) of 119.5 ft clearly shows that BCAS continuously underestimated the range. This indicates that the transponder reply delay of the intruder was shorter than the assumed 3 μs. However, this consistent bias does not have an effect on the model being developed.

EMPIRICAL DISTRIBUTION OF THE ERRORS. The histograms of the errors showed that the empirical distributions of the errors were unimodal and symmetrical. Using the method suggested by Hahn and Shapiro (reference 4), selected empirical distribution measures were computed to verify that the errors were normally distributed.

In general the error X_t at time t is given by

X_t = Theodolite measure - corresponding BCAS measure.

Let $\{X_t\}_{t=1}^{n}$ = a specified error set then

$$\overline{X} = 1/n \sum_{t=1}^{n} X_t$$

= mean error

and $$\mu_k = 1/n \sum_{t=1}^{n} (X_t - \overline{X})^k$$

= the biased estimate of the kth moment about the mean.

Then, the square of the standardized measure of skewness is

$$\beta_1 = \mu_3^2 / \mu_2^3$$

and the standarized measure of peakedness is

$$\beta_2 = \mu_4 / \mu_2^2$$

$\beta_1 = 0$ implies the distribution is symmetric. As a result, β_1 should be close to zero if the errors are normally distributed. For pure normal data $\beta_2 = 3$.

Hahn and Shapiro suggest the estimates of β_1 and β_2 are very sensitive to extreme observations in the sample, especially for sample sizes less than 200. The result of the analysis is summarized in table 2.

TABLE 2. β_1 AND β_2 VALUES OF THE ERRORS

Error	Sample Size	β_1	β_2
Own Altitude	63	0.14	1.5
Intruder Altitude	63	0.00	2.8
Range	56	0.18	2.7

The results showed that intruder altitude error and range error for the level flight data are approximately normally distributed. The own and intruder altitude information supplied to the BCAS are from the same source, i.e., the aircraft altitude encoders. Thus, one might expect the same distribution for own and intruder altitude errors. As can be seen in table 2, the β_2 estimate (1.5) for own altitude error is low for normal data. This is due to the quantization noise (the extreme observations at t = 26, 36, 43, and 51 seconds discussed in the previous subsection) present in the own altitude error data.

A review of figure 2, the range error data plot, showed extreme observations at t = 17, 18, 19, and 36. These extreme observations are due to missed reports in the raw flight test data. The range error values associated with the missed reports were replaced by the average of two previous observations. The standardized β_1 and β_2 measures for range errors shown in table 2 were then obtained.

SEQUENTIAL CORRELATION. The plots of the errors (figures 1 and 2) indicate that the error data are not independent from second to second. That is, the sequential errors appear to be time dependent or correlated. The null hypothesis, i.e., the sequential error deviations about the average error are sequentially uncorrelated, was tested using a run test (reference 5). A "run" in a sequence is a succession of elements with identical signs which is followed and preceded by elements of opposite signs or no elements at all. Thus, the number of runs in a sequence of error deviations is equal to the number of times the sign changes within the sequence plus 1. These hypotheses were strongly rejected at the 1-percent level. Table 3 presents the results of the analysis.

TABLE 3. RUN TEST RESULTS

Error Sequence	Number of Runs	1-Percent Critical Values: Lower	Upper	Results
Own Altitude	8	22	42	Reject H_o*
Intruder Altitude	7	22	42	Reject H_o
Range	12	22	42	Reject H_o

*Null Hypothesis, Ho: No sequential correlation.

Lindgren (reference 5) states the sufficient sample size for the run test is $n \geq 30$. Our sample size of 63 satisfies this requirement. The test confirmed that each sequence of errors is highly correlated. The number of runs of own altitude errors and intruder altitude errors was 8 and 7, respectively. This indicates that own altitude error and intruder altitude error may have similar sequential characteristics.

AUTOCORRELATIONS. The results of the run tests led to autocorrelation analysis of the level-flight error data. One of the assumptions involved in the computation of autocorrelation is that the mean, variance, and autocorrelations of the errors are independent of the absolute time. In general, tracking errors are known to be stationary (reference 6). Later in the report, it will be shown that the parameter estimates of the developed model satisfy the stationarity conditions.

Autocorrelation is a measure of sequential dependence of the error at time t on certain previous errors. The sequential position of the previous errors, on which the measure at time t depends, determines the autocorrelation lag. For example, autocorrelation for lag 1 is the measure of dependence of the error at time t (X(t)) on the error at time t-1 (X(t-1)). Similarly, the measure of dependence of the error X(t) on the kth preceding error X(t-k) is called the autocorrelation for lag k.

The autocorrelation for lag k can be computed by letting

n = the sample size
$X(t)$ = the observed value in the sample at time t

$$\text{then } \overline{X} = \frac{1}{n}\sum_{t=1}^{n} X(t) = \text{sample mean}$$

$$\text{and } C_k^2 = \frac{1}{n}\sum_{t=1}^{n-k} (X(t) - \overline{X})(X(t+k) - \overline{X})$$

$$= \text{sample autocovariance for lag } k \ (k = 1, 2, \ldots) \qquad (1)$$

Note that for lag o the measure is based on the current state only and

$$C_0^2 = \frac{1}{n}\sum_{t=1}^{n} (X(t) - \overline{X})^2 = \text{biased estimate of the variance}$$

Using (1) the sample autocorrelation for lag k, p_k is obtained as follows:

$$p_k = \frac{C_k2}{C_o2} \qquad k = 1, 2, \ldots \qquad (2)$$

The autocorrelations for lags 0 to 6 were computed and are presented in table 4. The autocorrelations for own altitude and intruder altitude errors do not differ significantly. Therefore, one could reason that the sequential dependence of own altitude error and sequential dependence of intruder altitude error are similar. This conclusion is plausible since the measuring process of both own and intruder altitude is similar.

TABLE 4. AUTOCORRELATIONS

Lag	Own Altitude Errors	Intruder Altitude Errors	Range Errors
0	1.000	1.000	1.000
1	0.892	0.895	0.681
2	0.743	0.763	0.420
3	0.563	0.595	0.197
4	0.368	0.426	0.059
5	0.177	0.282	-0.030
6	0.030	0.188	-0.093

PROPOSED MODEL

Box and Jenkins (reference 7) describe a dynamic process that can be used to characterize sequentially correlated time series data. It is called an autoregressive process. In this process, the current process deviation is a function of a fixed number (k) of previous process deviations. The fixed number k is called the order of the autoregressive process. A kth order autoregressive process could be written as follows:

Let

$X(t)$ = the value of the process at time t

μ = process mean

σ^2 = process variance

Then, $\underset{\sim}{X}(t) = X(t) - \mu$ is the process deviation or the unbiased value of the process at time t. Hence, $\underset{\sim}{X}(t)$ has mean = 0 and variance = σ^2. (Note: The notation $\underset{\sim}{X}(t)$ used here does not represent a vector.)

Let

Z_t = white noise at time t

and

$\left\{\phi_i\right\}_{i=1}^{k}$ = set of autoregressive parameters for kth order process

Then, the autoregressive process of order k ($k \geq 1$) could be expressed as

$$\underset{\sim}{X}(t) = \phi_1 \underset{\sim}{X}(t-1) + \phi_2 \underset{\sim}{X}(t-2) + \ . \ . \ . + \phi_k \underset{\sim}{X}(t-k) + Z_t; \ t \geq k + 1 \qquad (3)$$

Where Z_t, the white noise, is an identically distributed uncorrelated random variable. The distribution of Z_t will be discussed later.

In the kth order autoregressive process, the current process deviation $\underset{\sim}{X}(t)$ is a function of k previous process deviations $\underset{\sim}{X}(t-1)$, . . .$\underset{\sim}{X}(t-k)$. If the above process is a stationary process, the process mean, variance, and autocorrelations, P_i, i = 1, 2, . . . , are independent of absolute time.

The first-order autoregressive process could be written as

$$\underset{\sim}{X}(t) = \phi_1 \underset{\sim}{X}(t-1) + Z_t; \ t \geq 2 \ . \ . \ . \qquad (4a)$$

and is a stationary process if

$$-1 < \phi_1 < 1 \ . \ . \ . \qquad (4b)$$

For the first autoregressive process, the process deviation at time t, $\underset{\sim}{X}(t)$, depends only on the immediately preceding process deviation at time t-1, $\underset{\sim}{X}(t-1)$. Thus, the first-order autoregressive process is a Markov process. In prediction problems (considering $\underset{\sim}{X}(t)$ as the future process deviation and $\underset{\sim}{X}(t-1)$ as the current process deviation) the first-order autoregressive process is a process which has no memory. Higher order ($k \geq 2$) autoregressive processes are not Markov processes.

A second-order autoregressive process could be expressed as

$$\underset{\sim}{X}(t) = \phi_1 \underset{\sim}{X}(t-1) + \phi_2 \underset{\sim}{X}(t-2) + Z_t; \; t \geq 3 \; . \; . \; . \tag{5a}$$

where stationarity exists if the following are satisfied,

$$\begin{aligned} &\phi_1 + \phi_2 < 1 \\ &\phi_1 - \phi_2 > -1 \\ &-1 < \phi_2 < 1 \; . \end{aligned} \tag{5b}$$

DISTRIBUTION OF WHITE NOISE.

Equation 3 could be rewritten as

$$Z_t = \underset{\sim}{X}(t) - \phi_1 \underset{\sim}{X}(t-1) \; . \; . \; . - \phi_k \underset{\sim}{X}(t-k); \; t \geq k + 1 \tag{6}$$

Since the mean of $\underset{\sim}{X}(t)$'s = 0, the mean of $Z_t = 0$. The variance $\sigma^2_{Z_t}$ of Z_t is given by Gilchrist (reference 8)

$$\sigma^2_{Z_t} = (1 - \phi_1 P_1 - \phi_2 P_2 \; . \; . \; . - \phi_k P_k)\sigma^2 \tag{7}$$

where

P_i, i = 1, 2 . . . k are the autocorrelations of $\underset{\sim}{X}(t)$'s (equation (2)).

If $\underset{\sim}{X}(t)$'s are normally distributed, then Z_t's are independent and identically distributed normal random variables with mean zero and variance given by equation (7).

ESTIMATION OF AUTOREGRESSIVE PARAMETERS.

The maximum likelihood estimates, $\left\{\hat{\phi}_i\right\}_{i=1}^{k}$, of the autoregressive parameters can be established from the following recursive relation which was found in Jenkins and Watts (reference 9):

$$C_i^2 = \hat{\phi}_1 C_{i-1}^2 + \hat{\phi}_2 C_{i-2}^2 \; . \; . \; . + \hat{\phi}_i C_o^2 \, , \tag{8}$$

$$i = 1, 2, \; . \; . \; . k$$

when C_i^2 are computed according to equation (1).

For k = 2, that is the second-order autoregressive process, Jenkins and Watts suggest a better approximation for $\left\{\hat{\phi}_i\right\}_{i=1}^{2}$:

$$\hat{\phi}_1 = \frac{P_1 (1-P_2)}{1 - P_1^2} \tag{9a}$$

$$\hat{\phi}_2 = \frac{P_2 - P_1^2}{1 - P_1^2} \tag{9b}$$

DETERMINATION OF THE PROCESS ORDER.

The determination of the proper order of the autoregressive process is based on the fact that if an insufficient number of terms are used in the autoregressive model, the estimate of white noise variance, $\sigma^2_{Z_t}$, will be inflated by those terms which are not included. The minimum estimate of $\sigma^2_{Z_t}$ is obtained when the correct number of terms is included in the model.

From reference 9, the white noise variance ($\sigma^2_{Z_t}$) is estimated using the residual variance $S^2(k)$ where

$$S^2(k) = \frac{n-k}{n-2k-1} C_o^2 (1 - \hat{\phi}_1 P_1 - \hat{\phi}_2 P_2 \ . \ . \ . \ . - \hat{\phi}_k P_k) \tag{10}$$

The minimum estimate of $\sigma^2_{Z_t}$ is the minimum of the set

$$\left\{S^2(1),\ S^2(2)\ . \ . \ . \ ,\ S^2(k)\right\} = S^2_{min.}$$

As a result, the proper order, m, of the autoregressive process is that value of k such that $S^2(k) = S^2_{min}$.

MODEL FITTING

The results presented in model development showed that the level-flight data errors are autocorrelated, stationary, and normally distributed. Cohen and Richardson (reference 2) indicated that BCAS measurements and theodolite measurements were recorded once every second. Thus, one could consider each error (theodolite measurement - BCAS measurement) as a sample obtained by sampling at equal intervals of time (1 second) from a continuous time dependent process. The sample obtained in such a manner can be considered as a discrete time series.

The error sequences showed strong autocorrelations. A comparison of level-flight error data plots (figures 1 and 2) with climbing intruder error data plots (appendix B, figures B-1 and B-2) indicated that the errors are independent of their respective magnitudes of measurements; that is, as the measures of range and altitude increase the errors associated with these measures do not increase. These observations set favorable conditions for using autoregressive models.

ORDER OF THE AUTOREGRESSIVE MODELS.

The technique described previously was used to determine the order of the autoregressive models. The residual variances ($S^2(k)$, k=0, 1, 2, 3, 4) of own altitude error, intruder altitude error, and range error are presented in table 5. The minimum value of $S^2(k)$ in each case is identified. A first-order autoregression process (equation 4a) was found to be the appropriate model for the range error process, while second-order processes (equation 5a) were found to be adequate to represent own and intruder altitude errors. As expected, the own and intruder altitude error processes have the same order.

TABLE 5. ORDER SUFFICIENCY BASED ON $S^2(k)$

k	Range Error (ft^2)	Intruder Altitude Error (ft^2)	Own Altitude Error (ft^2)
0	9,152.663	588.896	1376.723
1	4,986.041*	119.159	286.078
2	5,402.392	116.870*	271.663*
3	5,566.567	136.535	310.101
4	5,687.115	144.654	335.725

*minimum value

PARAMETER ESTIMATES.

The autoregressive parameter estimates and the estimates of the white noise variances are shown in table 6. These estimates were computed according to the equations discussed earlier.

TABLE 6. MAXIMUM LIKELIHOOD ESTIMATES OF THE AUTOREGRESSIVE PARAMETERS AND VARIANCE OF Z_t

	Parameter Estimates		
Error Process	$\hat{\phi}_1$	$\hat{\phi}_2$	Variance of Z_t
Range	0.681	-	4,829.9
Own Altitude	1.122	-0.258	258.3
Intruder Altitude	1.066	-0.191	111.1

As indicated in the beginning of this report, these parameter estimates are based on a small sample size. The parameter estimates can be updated when more data are available.

CONFIDENCE REGIONS/INTERVALS OF PARAMETERS.

An approximate confidence region for kth order autoregressive parameters is discussed in Jenkins and Watts. Only two special cases, k=1 and k=2, need to be considered. They are the confidence interval of the first-order autoregressive parameter and confidence region of the second-order autoregresssive parameters (see figure 3).

For the first order process the 100 $(1-\alpha)$ percent confidence interval is given by the inequality

$$(\phi_1 - \hat{\phi}_1)^2 \leq \frac{S^2(1)\, F_{1,n-3}(1-\alpha)}{n\, C_0^2} \tag{11a}$$

where φ_1 is the unknown first order autoregressive parameter, $\hat{\phi}_1$ the estimated value of ϕ_1, $S^2(1)$ the residual variance, $F_{1,\ n-3}(1-\alpha)$ the 100 $(1-\alpha)$ the percentile of an F distribution with 1 and n-3 degrees of freedom, n the sample size, and C_o^2 is the biased estimate of the variance.

For the second-order process, the 100 $(1-\alpha)$ percent confidence region is given by the inequality

$$(\phi_1 - \hat{\phi}_1)^2 + 2\, P_1(\phi_1 - \hat{\phi}_1)\,(\phi_2 - \hat{\phi}_2) + (\phi_2 - \hat{\phi}_2)^2 \leq 2\, \frac{S^2(2)\, F_{2,\ n-5}(1-\alpha)}{n\, C_o^2} \tag{11b}$$

where ϕ_1 and ϕ_2 are the unknown second order autoregressive parameters and $\hat{\phi}_1$ and $\hat{\phi}_2$ are the respective estimates.

Figure 3 presents the 95-percent and 99-percent confidence regions of the own altitude error process parameters. The corresponding confidence regions of intruder altitude error process parameters are shown in figure 4. These regions are developed using relation (11b). In both figures the boundaries of the region within which the autoregressive process is stationary are the sides of a triangle with corners at (0,-1), (0,1), and (2,0) (see equation (5b)). These boundaries are identified in both figures. The points outside these boundaries are unacceptable, since they do not satisfy the stationarity conditions. The confidence regions of own and intruder altitude error process parameters overlap to a large extent. This is expected since they are of the same order and have nearly equal parameter estimates. An increase in the data base size would reduce the size of the confidence regions.

The 95-percent and 99-percent confidence intervals of the range error process parameter are respectively $0.494 < \phi_1 < 0.861$ and $0.432 < \phi_1 < 0.931$. Both confidence intervals are within the first-order autoregressive parameter stationarity condition (4b), $-1 < \phi_1 < 1$. All parameter estimates satisfy the respective stationarity conditions discussed earlier.

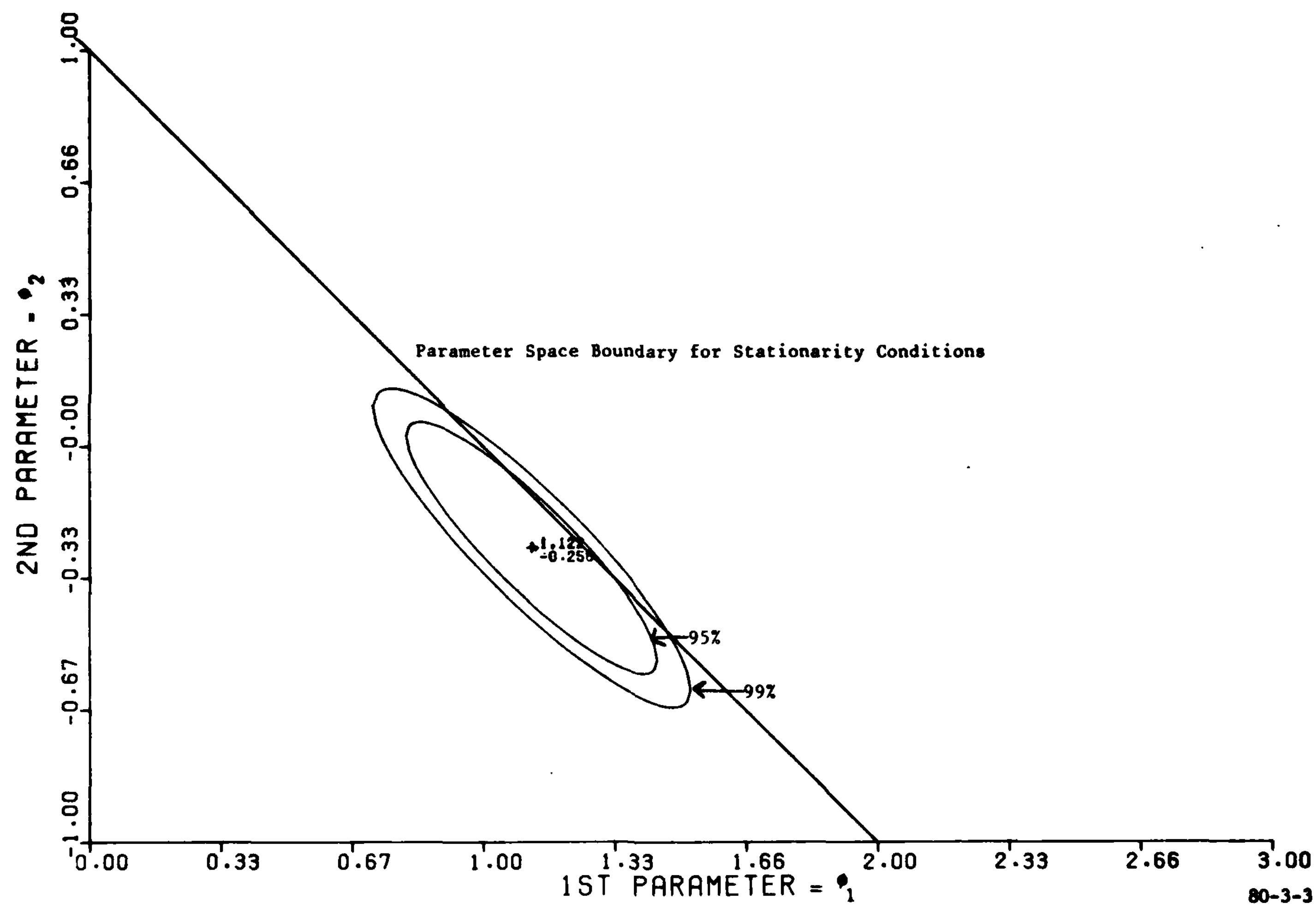

FIGURE 3. CONFIDENCE REGIONS FOR THE AUTOREGRESSIVE PARAMETERS FOR OWN ALTITUDE ERROR PROCESS

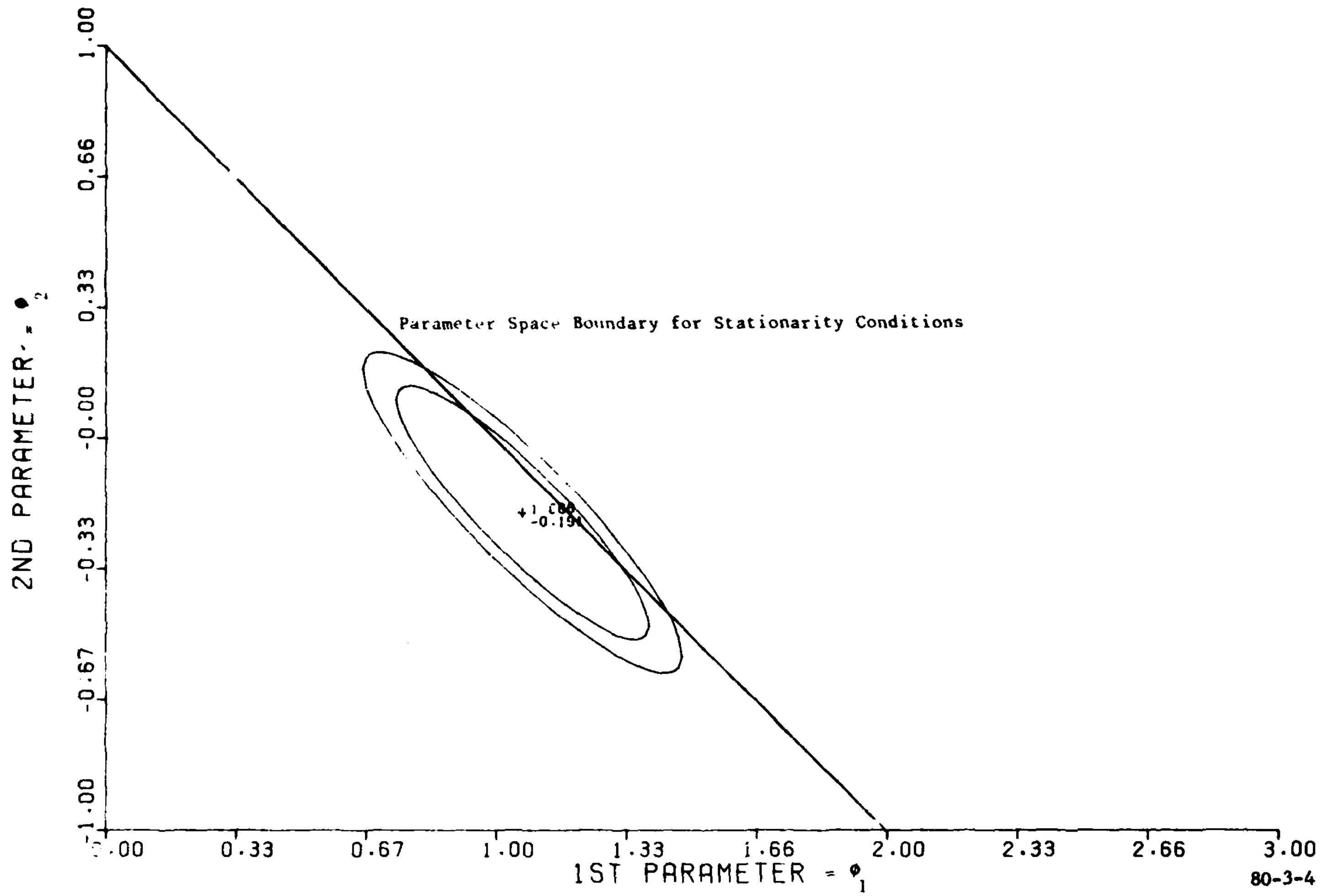

FIGURE 4. CONFIDENCE REGIONS FOR THE AUTOREGRESSIVE PARAMETERS FOR INTRUDER ALTITUDE ERROR PROCESS

SUMMARY OF MODEL FITTING.

The mean of the measurement errors is assumed to be zero. The bias observed in own altitude error is due to the altimeter bias. The modeling of altimeter bias and other types of altitude errors are not included in this report. The bias present in the range error is due to transponder reply delay. The range error process will be modeled without transponder delay errors at first, and then the model will be modified to include transponder reply delay errors.

ALTITUDE MEASUREMENT ERRORS. In previous sections, the similarities in own and intruder altitude error characteristics have been noted. They have the same order autoregressive models, nearly equal autocorrelations and parameter estimates. This was expected since the own and intruder altitude information supplied to the BCAS are similar. As a result, it would be appropriate to represent both own and intruder altitude errors with the same model.

For the limited data that were available, the analysis described in the preceding sections indicated that intruder altitude error data were more reliable than own altitude error data because they were not affected by mode C quantization noise or high bias due to experimental errors. For these reasons, the altitude measurement error process could be represented using the parameters developed from the intruder altitude error data. As a result, the active BCAS altitude measurement error process can be mathematically represented as

$$\mathcal{E}_A(t) = 1.066\ \mathcal{E}_A(t-1) - 0.191\ \mathcal{E}_A(t-2) + a_t;\ t \geq 3\ . . . \qquad (12)$$

where $\mathcal{E}_A(t)$ = the altitude measurement errors at time t, and a_t, the process white noise, is a normally distributed random variable with mean of zero feet, variance = 111.1 ft^2.

RANGE MEASUREMENT ERROR MODEL. The high bias present in the range error data is due to transponder reply delay. Hypothesizing that range measurement errors have zero mean and are normally distributed, the range measurement error process could be written as

$$\mathcal{E}_R(t) = 0.681\ \mathcal{E}_R(t-1) + b_t;\ t \geq 2 \qquad (13)$$

where $\mathcal{E}_R(t)$ = the range measurement error at time t, and b_t, the process white noise, is a normally distributed random variable with mean = zero feet and variance = 4,829.9 ft^2.

The bias in the range error represents half of the distance that could be covered at the speed of light during the transponder reply delay period. Since the transponder reply delay was assumed to be 3 μs in BCAS, the bias in the range error depends on the deviations of the transponder reply delay from the assumed 3 μs. Thus, the range error bias is given by

$$R_b = 1/2\ (983.516) \bullet (d-3) \text{ feet}$$

where 983.516 feet is the distance covered in 1 μs at the speed of light and d is a random variable (expressed in microseconds) having the distribution of transponder reply delays. From reference 10, d is uniformly distributed on the range [2.5, 3.5] μs.

The biased range error at time t, $E_R(t)$ is given by

$$E_R(t) = \mathcal{E}_R(t) + R_b$$

Substituting for $\mathcal{E}_R(t)$ from equation (13),

$$E_R(t) = 0.681\ \mathcal{E}_R(t-1) + R_b + b_t;\ t \geq 2$$

and

$$\mathcal{E}_R(t-1) = E_R(t-1) - R_b.$$

Hence

$$E_R(t) = 0.681\ E_R(t-1) + 0.319\ R_b + b_t;\ t \geq 2. \tag{14}$$

CONCLUSION

Analyses of two independent sets of Active Beacon Collision Avoidance System (BCAS) flight test data yielded very consistent results. Although the data base was limited, initial autoregressive error models were developed for the Active BCAS altitude and range measurement error processes. The consistency of the results of the autocorrelation analysis, the resulting autoregressive parameter estimates, and the process orders for the different data sets support the use of autoregressive modeling techniques. Changes in the active BCAS surveillance tracking functions could require that the autoregressive parameters be changed. It is unlikely that new data would cause the process orders to change. The models developed are based on the autocorrelation of the Active BCAS measurement errors. The techniques used in this analysis may be used for analysis of similar data. Based on statistical analysis of the available data, the altitude and range measurement errors may be represented by correlated Gaussion stationary time series. Appropriate models are presented in equations (12) and (14).

RECOMMENDATION

This study strongly supports the use of autoregressive processes to characterize the measurement errors in the Active BCAS algorithm evaluation. The major advantage of using autoregressive error model is that the high sequential correlation (autocorrelation) that exists in the Active BCAS measurement errors and their interactive effect on Active BCAS resolution can be characterized. The use of autoregressive error models provides a significant improvement in characterizing the Active BCAS input measurement process without increasing the model complexity.

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APPENDIX A

LEVEL-FLIGHT DATA

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THEODOLITE							BCAS							
TIME (SECONDS)	BCAS ALTITUDE Z_1	THREAT ALTITUDE Z_2	$\dot{Z}_1$	$\dot{Z}_2$	RANGE	$\dot{R}$	TRACK AGE (SINCE ESTABLISHMENT)	Z_1	Z_2	$\dot{Z}_1$	$\dot{Z}_2$	RANGE	$\dot{R}$
52178	2517	2927	-0.7998	6.0901	5.4589	-0.0784	1	2606	2881	5.0903	0.7080	5.4346	-0.0872
52179	2522	2922	5.1899	-4.8901	5.3724	-0.0865	2	2606	2881	5.0903	0.7080	5.3477	-0.0872
52180	2526	2921	4.7000	-1.7000	5.2883	-0.0841	3	2606	2881	5.0903	0.7080	5.2607	-0.0872
52181	2533	2919	6.5000	-1.8000	5.2081	-0.0802	4	2600	2888	5.0781	1.7700	5.1865	-0.0877
52182	2524	2914	-8.8899	-5.2998	5.1181	-0.0900	5	2594	2894	5.0659	2.4780	5.1006	-0.0872
52183	2530	2906	5.5999	-7.1101	5.0358	-0.0822	6	2594	2894	5.0659	2.8320	5.0146	-0.0868
52184	2531	2902	1.4001	-4.0901	4.9529	-0.0829	7	2594	2894	5.0659	2.8320	4.9277	-0.0868
52185	2529	2899	-2.5000	-3.7998	4.8696	-0.0833	8	2594	2894	5.0659	3.1860	4.8672	-0.0842
52186	2521	2899	-7.3101	0.2998	4.7843	-0.0853	9	2594	2900	5.0659	3.1860	4.7783	-0.0862
52187	2520	2902	-0.8000	3.5000	4.7015	-0.0828	10	2594	2906	5.0659	2.8320	4.6855	-0.0887
52188	2520	2905	-0.7900	3.0000	4.6225	-0.0790	11	2594	2906	5.0659	2.8320	4.5967	-0.0887
52189	2518	2913	-1.9099	8.0901	4.5383	-0.0842	12	2594	2906	5.0659	2.4780	4.5078	-0.0887
52190	2515	2909	-2.5000	-4.0901	4.4535	-0.0848	13	2594	2906	5.0659	2.1240	4.4248	-0.0866
52191	2517	2909	2.0000	-0.5000	4.3672	-0.0863	14	2594	2906	5.0659	1.7700	4.3457	-0.0838
52192	2511	2903	-5.8901	-6.1099	4.2822	-0.0850	15	2594	2906	5.0659	1.7700	4.2617	-0.0838
52193	2512	2899	0.8000	-3.3000	4.1992	-0.0830	16	2594	2906	5.0659	1.7700	4.1777	-0.0838
52194	2515	2889	2.7000	-10.3899	4.1169	-0.0823	17	2594	2931	5.0659	7.0801	4.1318	-0.0698
52195	2516	2878	1.0000	-11.0000	4.0305	-0.0864	18	2594	2925	5.0659	4.9561	4.0410	-0.0776
52196	2518	2875	2.3000	-2.7002	3.9491	-0.0814	19	2594	2925	5.0659	3.1860	3.9492	-0.0829
52198	2518	2875	2.3000	-2.7002	3.7787	0.8022	20	2594	2919	5.0659	0.0000	3.7666	-0.0877
52199	2511	2869	-5.5901	1.2002	3.6948	-0.0839	21	2594	2919	5.0659	0.0000	3.6787	-0.0877
52200	2510	2872	-1.1099	3.5000	3.6116	-0.0832	22	2569	2913	5.0171	-1.0620	3.5967	-0.0856
52201	2509	2873	-1.1899	1.3899	3.5283	-0.0833	23	2550	2906	4.9805	-1.7700	3.5137	-0.0846
52202	2506	2873	-3.0000	0.0000	3.4456	-0.0827	24	2531	2906	4.9438	-2.1240	3.4336	-0.0829
52203	2511	2877	5.2998	3.4099	3.3647	-0.0809	25	2519	2906	4.9194	-2.4780	3.3516	-0.0827
52204	2518	2888	6.5901	11.0901	3.2854	-0.0793	26	2506	2906	4.8950	-2.8320	3.2666	-0.0833
52205	2519	2891	1.2100	3.0000	3.2043	-0.0811	27	2525	2906	4.9316	-3.1860	3.1855	-0.0825
52206	2522	2893	2.7000	2.1101	3.1217	-0.0826	28	2544	2900	4.9683	-3.1860	3.1025	-0.0829
52207	2520	2890	-2.0000	-3.3101	3.0399	-0.0818	29	2563	2894	5.0049	-3.1860	3.0195	-0.0829
52208	2520	2892	0.3901	1.9099	2.9614	-0.0785	30	2575	2894	5.0293	-2.4780	2.9375	-0.0827
52209	2524	2887	3.6099	-4.8000	2.8793	-0.0821	31	2588	2894	5.0537	-2.1240	2.8613	-0.0801
52210	2524	2888	0.3901	1.0901	2.8000	-0.0793	32	2594	2894	5.0659	-1.7700	2.7813	-0.0801
52211	2529	2881	4.5000	-6.7000	2.7203	-0.0798	33	2600	2894	5.0781	-1.4160	2.7021	-0.0797
52212	2533	2878	4.2000	-3.0000	2.6402	-0.0801	34	2606	2894	5.0903	-1.0620	2.6191	-0.0807
52213	2531	2882	-1.5000	4.1099	2.5613	-0.0789	35	2606	2894	5.0903	-0.7080	2.5391	-0.0805
52214	2528	2881	-3.2000	-0.9099	2.4799	-0.0814	36	2606	2894	5.0903	-0.3540	2.4395	-0.0880
52215	2521	2885	-7.0901	3.3000	2.3998	-0.0801	37	2606	2894	5.0903	0.0000	2.3730	-0.0801
52216	2520	2890	-1.5000	5.0000	2.3198	-0.0800	38	2606	2894	5.0903	0.3540	2.2988	-0.0778
52217	2514	2887	-5.9099	-2.3000	2.2393	-0.0805	39	2581	2894	5.0415	0.7080	2.2236	-0.0766
52218	2512	2890	-2.0901	2.9099	2.1577	-0.0816	40	2556	2894	4.9927	1.0620	2.1436	-0.0780
52219	2511	2889	-0.8000	-1.7000	2.0782	-0.0795	41	2538	2894	4.9561	1.4160	2.0635	-0.0786
52220	2507	2890	-3.4099	1.4001	1.9969	-0.0812	42	2519	2894	4.9194	1.7700	1.9805	-0.0804
52221	2505	2899	-2.5000	9.0000	1.9153	-0.0816	43	2506	2894	4.8950	2.1240	1.9033	-0.0792
52222	2512	2901	7.3000	1.5999	1.8363	-0.0791	44	2494	2894	4.8706	2.1240	1.8242	-0.0792
52223	2518	2905	6.2000	4.0000	1.7558	-0.0804	45	2488	2894	4.8584	2.1240	1.7451	-0.0792
52224	2524	2913	5.1101	8.0901	1.6763	-0.0796	46	2513	2894	4.9072	2.1240	1.6660	-0.0792
52225	2525	2917	1.3899	3.9099	1.5946	-0.0817	47	2531	2894	4.9438	2.1240	1.5869	-0.0792
52226	2529	2921	3.6101	4.7000	1.5150	-0.0796	48	2550	2894	4.9805	2.1240	1.5078	-0.0792
52227	2526	2917	-2.3101	-4.7000	1.4297	-0.0852	49	2569	2894	5.0171	2.1240	1.4287	-0.0792
52228	2523	2914	-3.0901	-3.0999	1.3474	-0.0823	50	2581	2925	5.0415	8.1421	1.3418	-0.0819
52229	2517	2920	-5.7100	6.8899	1.2673	-0.0801	51	2594	2925	5.0659	6.3721	1.2588	-0.0823
52230	2513	2918	-4.7000	-2.2000	1.1879	-0.0793	52	2575	2925	5.0293	4.6021	1.1689	-0.0916
52231	2509	2917	-3.3901	-1.1899	1.1070	-0.0809	53	2556	2925	4.9927	2.8320	1.0820	-0.0899
52232	2503	2915	-6.6099	-1.9001	1.0243	-0.0828	54	2538	2919	4.9561	1.4160	1.0010	-0.0868
52233	2502	2922	-0.8901	6.5000	0.9445	-0.0798	55	2525	2919	4.9316	1.4160	0.9141	-0.0868
52234	2504	2923	1.7002	1.4001	0.8636	-0.0809	56	2513	2913	4.9072	0.3540	0.8369	-0.0839
52235	2505	2920	1.5000	-2.6101	0.7840	-0.0796	57	2506	2881	4.8950	-6.0181	0.6885	-0.1079
52236	2507	2921	2.0999	0.9099	0.7051	-0.0789	58	2500	2881	4.8828	-4.6021	0.6338	-0.0882
52237	2511	2930	4.2000	8.2002	0.6266	-0.0785	59	2494	2881	4.8706	-3.1860	0.5762	-0.0771
52238	2511	2929	-0.7000	-0.8101	0.5481	-0.0785	60	2494	2881	4.8706	-1.7700	0.5117	-0.0722
52239	2516	2919	5.2000	-9.8000	0.4641	-0.0840	61	2494	2888	4.8706	-0.7080	0.4424	-0.0710
52240	2519	2913	2.7002	-6.3899	0.3840	-0.0802	62	2494	2894	4.8706	0.0000	0.3691	-0.0718
52241	2524	2910	5.5000	-2.2002	0.3068	-0.0772	63	2494	2894	4.8706	0.3540	0.2930	-0.0734

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FIGURE A-1. LEVEL-FLIGHT DATA

APPENDIX B

CLIMBING INTRUDER DATA ANALYSIS

Numerous targets of opportunity that were climbing or descending were tracked in the Beacon Collision Avoidance System (BCAS) flight tests at NAFEC. However, the data do not contain theodolite measurements (true measurements). The determination of the errors required additional assumptions. The MITRE Corporation provided a set of data for a climbing intruder in which the vertical climb rate and range rate were nearly constant. The set consisted of 46 seconds (46 points) of data. It was hoped that the data would allow us to determine if the vertically maneuvering intruder measurement error characteristics are drastically different from the error characteristics of the level-flight intruder.

Since theodolite "true" measurements were not available, the "true" measurements were computed assuming a constant climb rate. The errors were calculated by subtracting BCAS measurements from true measurements.

The minimum slant range to the intruder (1.64 nautical miles (nmi)) occurred between the 28th and 29th second of data. A constant closure rate was assumed for the first 28 seconds of data, and a constant separation rate was assumed for the last 18 seconds of data. The range error is the difference between the range computed using a constant closure rate and the BCAS slant range, and the range computed using a constant separation rate and the BCAS range for the remaining 18 seconds.

Figure B-1 presents the plot of climbing intruder altitude error as a function of time. The plot of range error as a function of time is shown in figure B-2. The error averages are indicated in both figures.

TEST ORGANIZATION.

The objective of this analysis was to justify the models. Preliminary analysis such as the run test, correlation analysis, and normality checks were conducted on the data. The results of the analyses are not included in this report. They are consistent with the results of the level-flight data analysis.

The means, variances, order of the models, and parameter estimates are presented here. The discrepancies, if any, with the results of the level flight data analysis results are explained.

MEAN AND VARIANCE.

The means and variances of the errors are presented in table B-1. A comparison with the values presented in table 1 shows that the variation in climbing intruder range error is four times the corresponding variation in level-flight data. This is probably due to the constant rate assumption and/or the small sample size. The average of range error, -63.5 feet, is due to a larger than B-1 expected transponder delay (> 3 microseconds), and thus BCAS overestimated the range. The mean of altitude error, 7.8 feet, is comparable to the average of the level-flight intruder altitude error average, 0.3 feet; the standard deviations, 21.1 feet and 24.1 feet are also nearly equal.

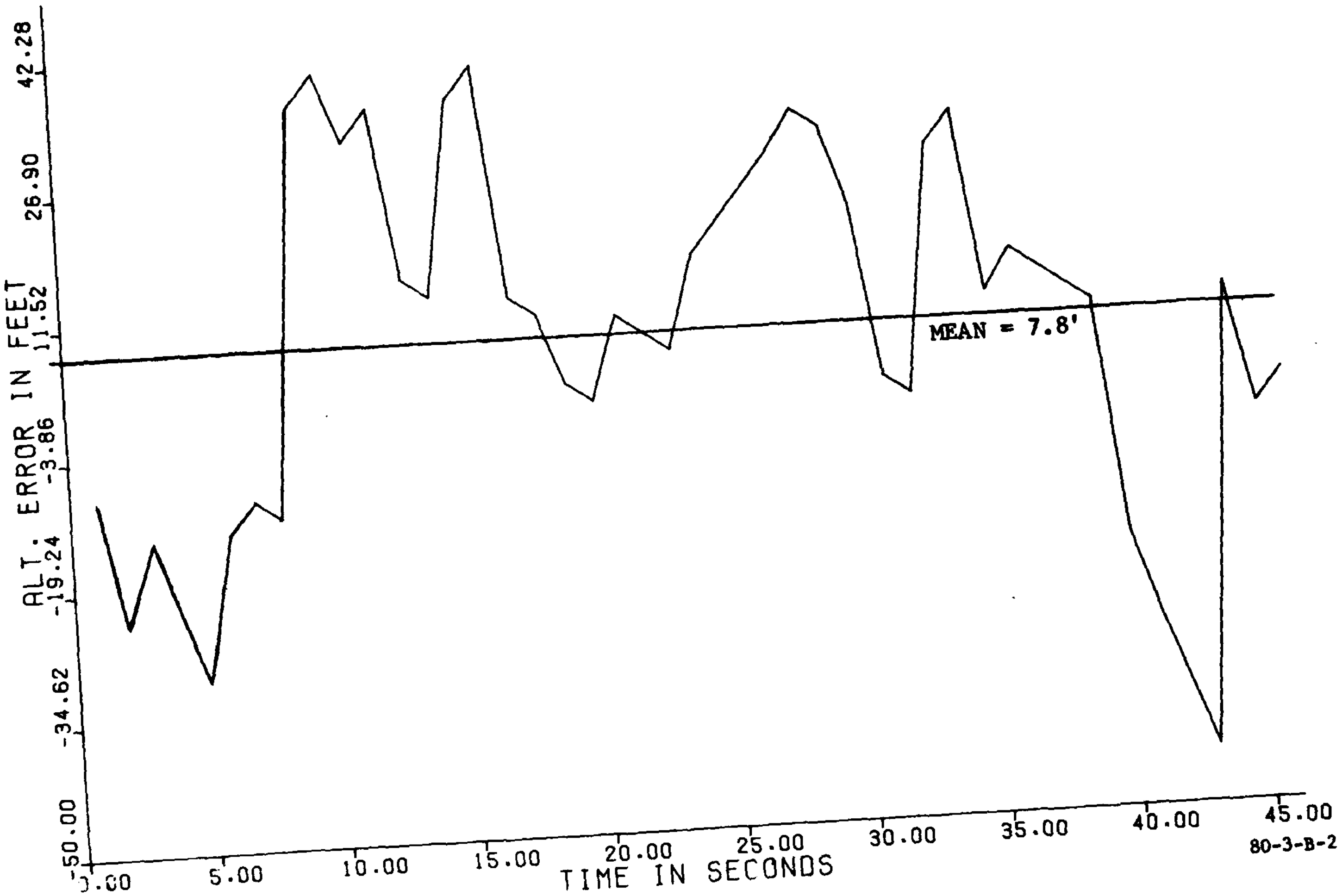

FIGURE B-1. SEQUENTIAL ALTITUDE ERROR FOR CLIMBING INTRUDER

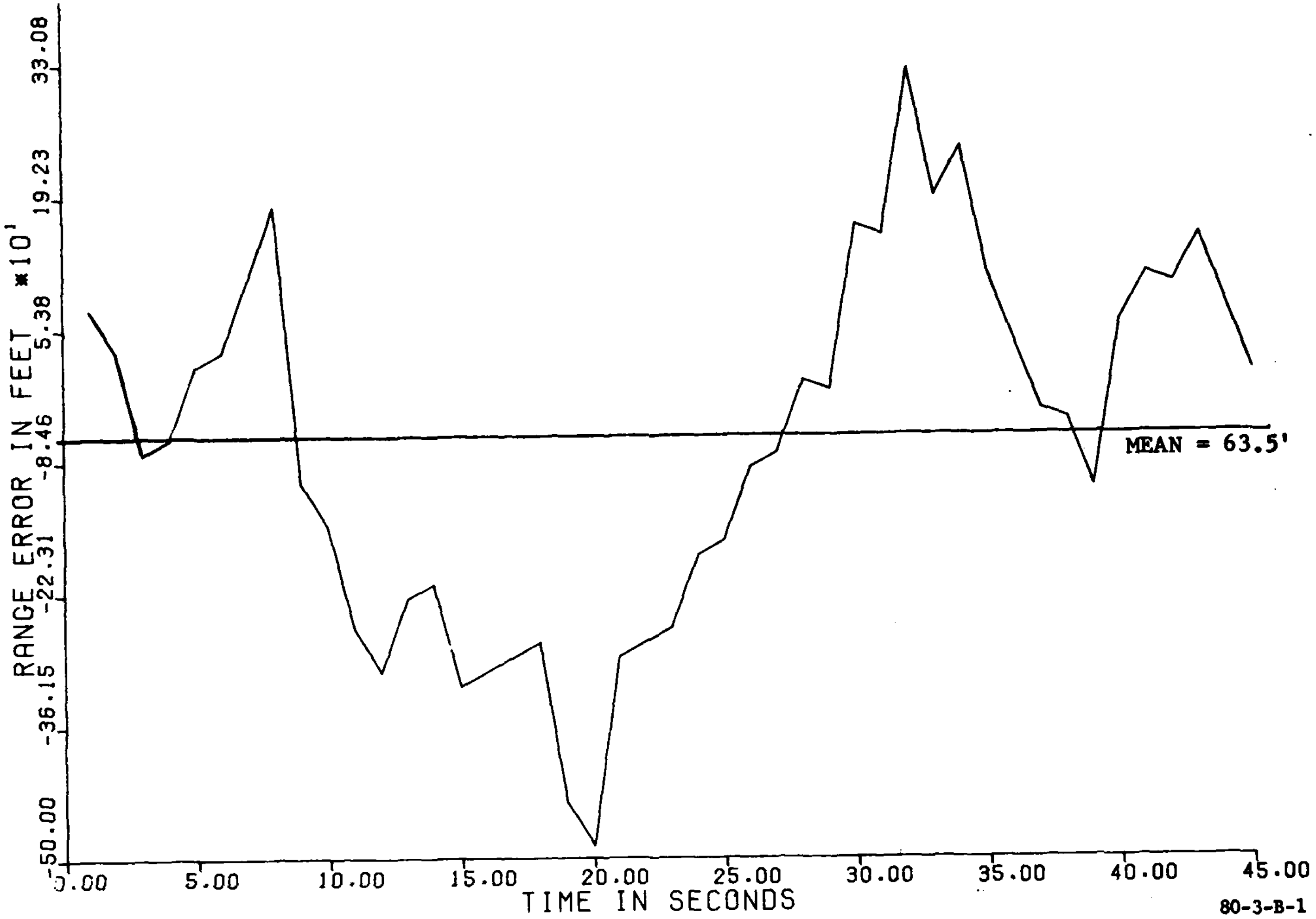

FIGURE B-2. SEQUENTIAL RANGE ERROR FOR CLIMBING INTRUDER

TABLE B-1. MEAN AND VARIANCE

Error	Average (ft)	Sample Variance (ft^2)	Standard Deviation (ft)
Intruder Altitude	7.8	445.4	21.1
Range	-63.5	35363.5	188.1

PARAMETER ESTIMATES FROM CLIMBING INTRUDER DATA.

The maximum likelihood estimates of the autoregressive parameters and white noise variance are presented in table B-2. In the case of range error, both estimates, parameter and white noise variance, are much higher than the respective estimates obtained from level flight data. Higher estimates were expected because of the higher variance of the climbing intruder range error data.

TABLE B-2. MAXIMUM LIKELIHOOD ESTIMATES OF THE AUTOREGRESSIVE PARAMETERS AND VARIANCE OF Z_t

Error Process	Parameter Estimates $\hat{\phi}_1$	$\hat{\phi}_2$	Variance of Z_t
Range	0.888	-	27785.1
Altitude	0.813	-0.104	98.9

For the climbing intruder altitude data, the first autoregressive parameter estimate is lower, and the second parameter estimate is higher than the corresponding estimates for level-flight data. It is very likely that additional correlation is introduced in the error data by assuming a constant climb rate.

ORDER OF THE MODELS.

Orders of the autoregressive models were computed using the technique described earlier. The results are summarized in table B-3.

TABLE B-3. ORDER SUFFICIENCY OF CLIMBING INTRUDER DATA BASED ON $S^2(K)$

k	Range Error	Intruder Altitude Error
0	36,167	455
1	7,794*	213
2	7,993	216
3	9,199	216
4	—	213

*Minimum Value

Residual variance, $S^2(k)$, showed unique minimum at $k = 1$ in the case of range error, indicating a first-order autoregressive process is adequate to represent the range error data. In the case of intruder, altitude error, $S^2(k)$, remained almost constant for $1 \leq k \leq 3$, which indicated that a second-order autoregressive process was sufficient. The results are consistent with the results of the level-flight data analysis.